Linear theory of plasma wakes

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(Received 11 January 2000)

A test particle immersed in a plasma produces a wake when at least one plasma species is drifting. The present work presents a new approach to the study of plasma wakes. The solution to the problem is found in terms of a spherical harmonics expansion. The solution is studied for conditions typical in dusty plasma experiments: the ions stream relative to the test particle, while the electrons have zero average velocity. The results confirm previous findings obtained with theories based on the Fourier transform method. The present method is characterized by two new aspects. First, the wake field is studied as a function of the ion to electron temperature ratio. Second, the present theory is developed fully in the real space, without introducing the Fourier transform, and can be more naturally extended to include nonlinear effects and to include the collisions between dust particles and plasma species.

PACS number(s): 52.25.Zb, 52.75.−d, 52.65.Kj, 52.25.Dg

I. INTRODUCTION

The study of plasma wakes is a classic problem in plasma physics with applications ranging from fusion experiments and laser–plasma interaction [1] to dusty plasma [2]. Recently the subject has received particular attention, for its relevance to studies in dusty plasma crystals [3]. Dusty plasma crystals form in glow discharge reactors at the edge of the sheaths that surround the electrodes. The ions are accelerated to sound speed in the presheath and travel even faster in the sheath itself. Therefore, depending on the precise position of the dust particles, near sonic or even supersonic ion flow conditions are found around the dust particles. In such situations, wakes can develop, altering considerably the usual Debye shielded potential [2].

Traditionally, the study of plasma wakes has been treated using a linear approach where a point charge is assumed to perturb a plasma described by the linearized Boltzmann–Poisson system. The classic solution [4] uses the Landau method (equivalent to the application of the Fourier and Laplace transforms). More recently, the problem has been reconsidered, focusing on the specific conditions found in the reactors used for dusty plasma crystal experiments [5].

However, recent works have suggested that standard methods valid in dust-free plasmas are not suitable for dusty plasma conditions [6]. The defining property of a dusty plasma is the presence of currents of ions and electrons collected by the dust particles. Such a process alters the distribution function of the plasma species and could have an important effect on plasma wakes. Indeed, even in the absence of any relative drift, it has been shown that the orbit motion limited (OML) theory that includes the effects of the currents collected by the dust yields different results from the classic Debye shielding [7]. But even the OML is not completely accurate [8,9]. A complete self-consistent simulation approach has shown that the actual plasma wake is indeed different from previous theoretical predictions and in much better agreement with experimental results [10,11].

In the present work, a new theoretical approach is proposed. The method is based on the representation of the angular dependence of the particle distribution functions using spherical harmonics. The approach is widely used in glow discharge studies [12] and more recently it has been applied also to dusty plasma [13].

The advantage of this approach is twofold. First, the linear study of plasma wakes avoids the analytical intricacies of the usual Landau approach and in particular the singularities of the integrals. As a consequence, the results of the present model are easy to obtain and unambiguous. Second, the relative simplicity of the final model equations allows the extension of the present model to include more realistic processes. In particular, it allows the inclusion of the sheath electric field and of the charging currents to the dust particles.

This paper is organized in four sections, besides the Introduction. Section II describes the method used to derive the model equations, focusing on the spherical harmonics expansion and on the other approximations introduced. Section III discusses the properties of the model equations and the solution procedure. Section IV presents the results obtained for different plasma parameters (Mach number of the ion flow, ion to electron temperature ratio, collision frequency between ions and neutrals). Finally, Sec. V summarizes the results.

II. MATHEMATICAL MODEL

We consider the interaction of a plasma with a test particle immersed in it. The plasma is in general composed by several species (only one of which is electrons). The derivation assumes that the plasma species are not magnetized and that electromagnetic waves can be neglected.

Under these conditions, the kinetic description of the system is formulated in terms of the distributions of the plasma species s in the phase space $f_s \in \mathbb{R}^6 \times [0,T[$ and of the electrostatic potential $\varphi \in \mathbb{R}^3 \times [0,T[$

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$$\frac{\partial f_s}{\partial t}+\mathbf{v}\cdot\frac{\partial f_s}{\partial \mathbf{x}}-\frac{q_s}{m_s}\frac{\partial \varphi}{\partial \mathbf{x}}\cdot\frac{\partial f_s}{\partial \mathbf{v}}=\mathrm{St}\{f_s\}, \quad (1)$$

$$\mathbf{E}_0\nabla^2\varphi=-\sum_s q_s\int f_s\, d\mathbf{v}, \quad (2)$$

where $\mathrm{St}\{f_s\}$ is the collision operator. In the conditions typical for the formation of dusty plasmas, the collisions among plasma particles are negligible, and the collision operator should only include collision of plasma particles with neutrals and with the test particle. In the present work, the collision operator can have a very general form [14].

The model equations (1) are solved in three steps. In the first step, the equations are linearized around the unperturbed plasma conditions chosen to represent the plasma without the test particle. The physical intuition suggests that the introduction of a test particle perturbs the plasma. If the test particle is sufficiently small (in terms of the Debye length) the perturbations are expected to be small and liable to be treated linearly. This approximation is widely used by the theories presented in the literature [4,5]. However, if the test particle represents the typical dust particles found in experiments, the charge of the test particle can be 3 orders of magnitude larger than the electron charge. For this reason, the results of any linear theory require validation against experiments and nonlinear simulations [10].

Within these limitations, the plasma state is described as a perturbation induced by the immersed object on an otherwise unperturbed system. In the unperturbed state the plasma species are assumed to be distributed according to a drifting Maxwellian

$$f_{0s}=\frac{n_{0s}}{v_{th,0s}^3(2\pi)^{3/2}}\exp\left(-\frac{(\mathbf{v}-\mathbf{v}_{0s})^2}{2v_{th,0s}^2}\right), \quad (3)$$

where the species-dependent drift speed $\mathbf{v}_{0s}$, thermal velocity $v_{th,0s}$, and density n_{0s} are chosen to represent the experimental conditions.

The perturbed state is characterized by a perturbed electrostatic potential φ_1 (and electric field $\mathbf{E}_1$), and perturbed species distributions f_{1s}. The linearized Boltzmann equation is

$$\frac{\partial f_{1s}}{\partial t}+\mathbf{v}\cdot\frac{\partial f_{1s}}{\partial \mathbf{x}}-\frac{q_s f_{0s}}{m_s v_{th,0s}^2}\mathbf{E}_1\cdot(\mathbf{v}-\mathbf{v}_{0s})=\mathrm{St}\{f_s\}, \quad (4)$$

where subscript 0 (1) refers to unperturbed (perturbed) quantities.

In the second step of the derivation, the functional dependence upon the direction of the velocity variable is represented using spherical harmonics. The velocity vector is represented using a modulus u and a unitary vector $\mathbf{\Omega}$

$$\mathbf{v}=u\mathbf{\Omega}+\mathbf{v}_{0s} \quad (5)$$

and the Boltzmann equation becomes

$$\left(\mathbf{\Omega}+\frac{\mathbf{v}_{0s}}{u}\right)\cdot\frac{\partial f_{1s}}{\partial \mathbf{x}}-\frac{q_s f_{0s}}{m_s v_{th,0s}^2}\mathbf{\Omega}\cdot\mathbf{E}_1=\frac{1}{u}\mathrm{St}\{f_s\}. \quad (6)$$

A general method for the solution of the linear Boltzmann equation is the spherical harmonics expansion [15]. Truncating at the first order, the expansion can be written as

$$f_{1s}(u,\mathbf{\Omega})=A_s(u)+\mathbf{\Omega}\cdot\mathbf{B}_s(u), \quad (7)$$

where $A_s(u)$ and $\mathbf{B}_s(u)$ are the new variables to be determined in order to solve the system. An integration in the velocity space reveals the physical meaning of the two new variables in terms of the perturbation of the ion density and of the ion current

$$n_{1s}=4\pi\int A_s u^2\, du, \quad (8)$$

$$\mathbf{J}_{1s}=\frac{4\pi}{3}\int \mathbf{B}_s u^3\, du. \quad (9)$$

Inserting the expansion Eq. (7) into Eq. (6) and assuming, without loss of generality, that the reference frame has the z axis directed along the drift velocity $\mathbf{v}_{0s}$, it follows that

$$\left(\mathbf{\Omega}+\frac{\mathbf{z}v_{0s}}{u}\right)\cdot\frac{\partial}{\partial \mathbf{x}}(A_s+\mathbf{\Omega}\cdot\mathbf{B}_s)-\frac{q_s f_{0s}}{m_s v_{th,0s}^2}\mathbf{\Omega}\cdot\mathbf{E}_1=\frac{1}{u}\mathrm{St}\{f_s\}. \quad (10)$$

Equation (10) is solved with a projection method in the direction sphere $\mathbf{\Omega}$. Integrating Eq. (10) in the direction sphere $\mathbf{\Omega}$ yields

$$\frac{\partial A_s}{\partial_z}=-\frac{u}{3v_{0s}}\nabla\cdot\mathbf{B}_s+\frac{1}{4\pi v_{0s}}\int \mathrm{St}\{f_s\}d\mathbf{\Omega}. \quad (11)$$

Integrating Eq. (10) multiplied by $\mathbf{\Omega}$ yields

$$\frac{\partial \mathbf{B}_s}{\partial_z}=-\frac{u}{v_{0s}}\nabla\left(A_s+\frac{q_s}{m_s v_{th,0s}^2}f_{s0}\varphi_1\right)+\frac{3}{4\pi v_{0s}}\int \mathrm{St}\{F_s\}\mathbf{\Omega}\, d\mathbf{\Omega}. \quad (12)$$

The integrals of the collision operator can be calculated under a very general hypothesis [14,15]. In the following, the effect of plasma currents to the immersed particle is neglected, as customary in classic plasma wake theories [4]. This approximation allows a direct comparison with the previous literature. Future work will include collisions with the test particle to include effects that might be significant for dusty plasma experiments [6,7,10].

With this approximation and assuming further that no ionization or recombination collisions are present, the first integral [in Eq. (11)] vanishes

$$\int \mathrm{St}\{f_s\}d\mathbf{\Omega}=0. \quad (13)$$

The second integral [in Eq. (12)] can be expressed in terms of the velocity dependent momentum transfer collision frequency $\nu(u)$ [12]:

$$\int \mathrm{St}\{f_s\}\mathbf{\Omega}\, d\mathbf{\Omega}=-\nu\mathbf{B}_s. \quad (14)$$

Equations (11),(12) with the collision integrals, Eqs. (13),(14) form a self-consistent closed system to describe a plasma species. Note that Eqs. (11),(12) are the linearized version of the standard two-term Boltzmann model used to study gas discharges [12].

The two equations (11),(12) can be easily combined in just one equation for the single variable A_s, by taking the divergence of the second equation and the z derivative of the first and substituting

$$\frac{\partial^2 A_s}{\partial z^2}=\frac{1}{3}\left(\frac{u}{v_{0s}}\right)^2\nabla^2\left(A_s+\frac{q_s}{m_s v_{th,0s}^2}f_{s0}\varphi 1\right)-\frac{\nu}{v_{0s}}\frac{\partial A_s}{\partial z}. \tag{15}$$

Equation (15) summarizes the linearized Boltzmann equation for the ion population in the approximation of first order spherical harmonics expansion. The study of plasma wakes could be based on the model equation (15), but in the present work a further simplification is introduced to arrive to a fluid model.

In the third step of the derivation, the model equations are integrated in the amplitude of the velocity vector u, leading to

$$\frac{v_{0s}^2}{v_{th,1s}^2}\frac{\partial^2 n_{1s}}{\partial z^2}=\nabla^2\left(n_{1s}+\frac{q_s}{m_s v_{th,1s}^2}n_{0s}\varphi 1\right)-\frac{\overline{\nu}v_{0s}}{v_{th,1s}^2}\frac{\partial n_{1s}}{\partial_z}, \tag{16}$$

where $v_{th,1s}^2=4\pi\int Au^4du/3n_{1s}$ is the thermal velocity of the perturbed distribution and $\overline{\nu}$ the average collision frequency

$$\overline{\nu}=\frac{4\pi}{n_{1s}}\int\nu(u)Au^2\,du. \tag{17}$$

The closure of the model equations is obtained assuming that the thermal velocity of the perturbed distribution $v_{th,1s}$ is governed by a linearized equation of state

$$v_{th,1s}^2=\gamma v_{th,0s}^2, \tag{18}$$

where $\gamma=5/3$ corresponds to an adiabatic transformation and $\gamma=1$ to an isothermal transformation.

Equation (16) is the final equation for the species density used in the subsequent sections to study the plasma shielding of a test particle.

III. GENERAL PROPERTIES AND SOLUTION PROCEDURE

Equation (16) governs the perturbation of the plasma species in response to the introduction of a test particle. The condition of the unperturbed ambient plasma enters the equation via the species density, thermal speed and drift velocity. The model equations can be solved for any kind of ambient plasma. In the present work, the attention is focused on the typical conditions found in dusty plasma experiments: the ions are drifting at a velocity exceeding the ion thermal speed, but the electrons have no drift velocity. Such conditions are an accurate description of the sheath regions where the dust particles collect [3]. Within that assumption, it follows that Eq. (16) for the electrons can be simplified to the usual linearized Boltzmann relationship. The complete model for the system becomes

$$\frac{v_{0i}^2}{v_{th,0i}^2}\frac{\partial^2 n_{1i}}{\partial z^2}=\nabla^2\left(\gamma n_{1i}+\frac{qi}{m_i v_{th,0i}^2}n_0\varphi_1\right)-\frac{\overline{\nu}v_{0i}}{v_{th,0i}^2}\frac{\partial n_{1i}}{\partial z},$$

$$n_{1e}=\frac{e}{m_e v_{th,0e}^2}n_0\varphi_1, \tag{19}$$

$$\epsilon_0\nabla^2\varphi_1=e(n_{1e}-n_{1i})+q_t\delta(\mathbf{x}-\mathbf{x}_t),$$

where $q_t(\mathbf{x}_t)$ is the charge (position) of the test particle.

The system Eq. (19), can be written more conveniently introducing the normalized potential and ion density

$$\Phi=\frac{\epsilon_0\phi_1\lambda_{De0}}{q_t},\ N=\frac{en_{1i}\lambda_{De}^3}{q_t}. \tag{20}$$

Using the definitions above and eliminating the electron density from the second and third equation, it follows readily that

$$\frac{\partial^2 N}{\partial\zeta^2}=\frac{1}{M^2}\nabla_{\boldsymbol{\xi}}^2(\Theta N+\Phi)-R\frac{\partial N}{\partial\zeta},$$

$$\nabla_{\boldsymbol{\xi}}^2\Phi=(\Phi-N)+\delta(\boldsymbol{\xi}-\boldsymbol{\xi}_t), \tag{21}$$

where $\Theta=\gamma T_i/T_e$, $R=\overline{\nu}\lambda_{De}/v_{0i}$, and M is the Mach number. The differential operators are written in terms of the normalized spatial variable $\boldsymbol{\xi}=(\xi,\eta,\zeta)=\mathbf{x}/\lambda_{De}$.

The solution of system (21) depends on three parameters only: the Mach number related to the speed of the ion flow; the temperature ratio Θ modified by the exponent γ of the equation of state; and parameter R related to the collision frequency between ions and neutrals. The advantage of the formulation (21) in terms of normalized quantities is to enucleate such dependence.

Some general properties of the system Eq. (21) are crucial in determining the properties of its solutions. First, the equation for the potential [second part of Eq. (21)] is elliptic but the equation for the ion density [first part of Eq. (21)] is hyperbolic (i.e. a wave equation) when $M^2/\Theta>1$. Second, the problem is well posed only if the boundary conditions are assigned according to the nature of the equations. The elliptic equation for Φ is supplemented with the usual Dirichlet condition, $\Phi=0$ on the boundary ∂V of the system. The hyperbolic equation for N, instead, requires initial conditions along ζ (that here is the time-like coordinate of a wave equation): $N=0$ and $\partial N/\partial\zeta=0$ at the inflow boundary.

System (21) can be solved numerically. Some properties of the solution could be obtained using analytical techniques based on the characteristics method [16]. However, the simplicity of the numerical solution suggests its preferred use. Standard techniques are used.

First, each equation is discretized. Cylindrical coordinates (ρ,θ,ζ) centered on the dust particle are used with the axis of symmetry directed along the flow direction. The symmetry of the system allows one to drop any θ dependence. The elliptic equation is discretized in ρ and ζ using the standard finite difference approximation to the Laplacian operator. The hyperbolic equation is discretized in the spatial coordinate ρ using a finite difference approximation of the second

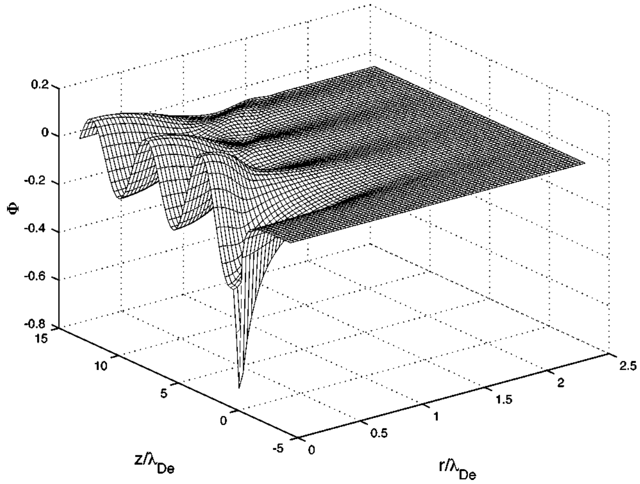

FIG. 1. Perturbation of the normalized potential Φ induced by a negative unitary charge immersed in a plasma with $\Theta=0.01$ and with ions streaming at $M=0.8$.

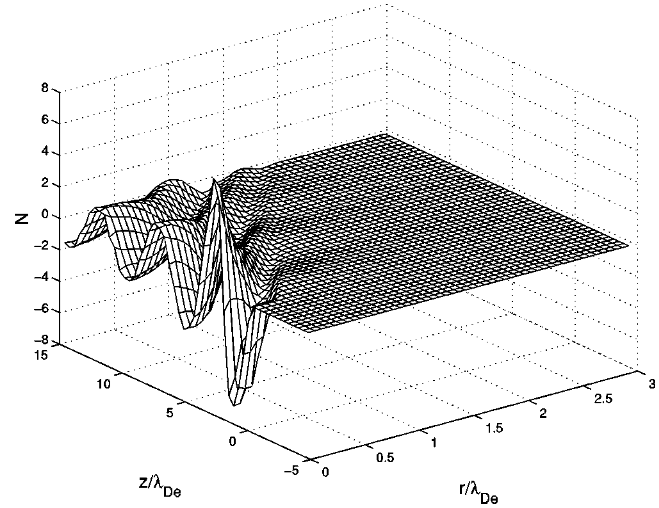

FIG. 2. Perturbation of the normalized ion density N induced by a negative unitary charge immersed in a plasma with $\Theta=0.01$ and with ions streaming at $M=0.8$.

derivative; the time-like coordinate ζ is discretized instead, using the leap-frog method. [17] The leap-frog method is explicit, requiring a dual grid method along the time-like coordinate (i.e., more points in ζ are used when solving the hyperbolic equation).

Second, the discretized equations are solved using a block Gauss–Siedel iteration method: the equation for Φ is solved using N from the previous iteration, next the equation for N is solved using the new guess for Φ. The method is found to converge in just a few iterations. In the cases described below, the norm of the variation of the unknown between two iterations is found to decrease by 3 orders of magnitude in 20 iterations.

The convergence of the spatial discretization has been verified also. In the numerical solutions shown in Sec. IV, the spatial grid used is 150×150 for a system size $2.5\lambda_{De}$ and $20\lambda_{De}$ in the radial and vertical direction, respectively.

IV. WAKE STRUCTURE

The theory presented above has been used to study plasma wakes in different plasma conditions, varying the temperature ratio between the plasma species, varying the Mach number of the ion flow, and varying the ion-neutral collision rate.

Figure 1 (electrostatic potential) and Fig. 2 (ion density) show the complete three dimensional (3D) structure of the plasma wake for $M=0.8$ and $\Theta=10^{-2}$. The azimuthal symmetry of the problem allows us to represent the electrostatic potential and the ion density as a function of the vertical distance ζ and of the radius ρ measured from the axis centered on the dust particle and directed along the ion flow.

Three features are considered below:

First, the ion density show the presence of a Mach cone with angle $\sin^{-1}(\sqrt{\Theta}/M)$ with respect to the flow direction. However, the coupling with the elliptic equation for Φ renders the Mach cone diffuse and allows for information transmission outside the Mach cone.

Second, an oscillatory wake is present downstream. The wake is only present inside the Mach cone and its amplitude decreases (even in the absence of collisions) as the wake broadens downstream.

Third, in the direction normal to the ion flow, the potential conserves its classic Debye shielding form, but with a decay rate very close to the electron Debye length, as shown in previous simulations [10].

The last two effects are considered further in Secs. IV A and IV B.

A. Oscillatory wake

The oscillatory wake shown in Figs. 1 and 2 is a well known result of linear theories. Previous works [18], valid in the limit of zero ion temperature, have found that the wavelength Λ of the wake field is

$$\Lambda/\lambda_{De}=2\pi M. \tag{22}$$

Figures 3 and 4 compare the plasma wake along the ζ axis for different values of the Mach number M and of the temperature ratio Θ. As the Mach number M is increased, the

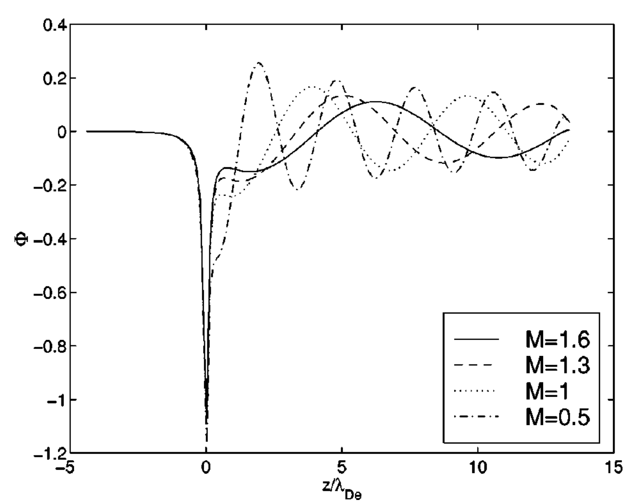

FIG. 3. Perturbation of the normalized potential Φ along the ζ axis for $\Theta=0.01$ and for different Mach numbers.

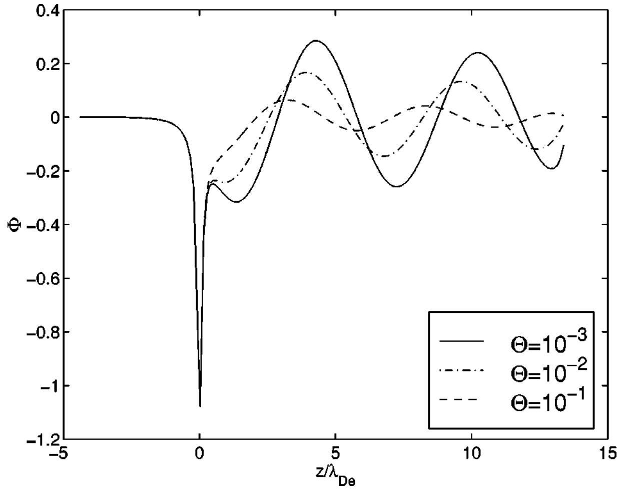

FIG. 4. Perturbation of the normalized potential Φ along the ζ axis for $M=1$ and for different values of the temperature ratio Θ.

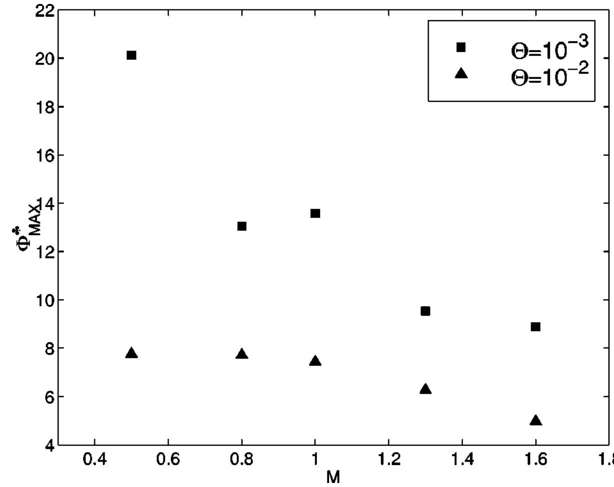

FIG. 6. Amplitude $\Phi^{\clubsuit}_{MAX}$ of the largest fourier mode of the potential as a function of the Mach number for two values of the temperature ratio Θ.

wavelength increases but the amplitude of the wave decreases. An increase of the temperature ratio Θ has the effect to decrease both amplitude and wavelength.

Figure 5 shows the wavelength of the wake field obtained from the numerical solution of the present model, Eq. (21), for various temperature ratios and Mach numbers. Clearly, the present model yields the classic expression, Eq. (22), in the limit of zero ion temperature. As the ion temperature is increased, the wavelength decreases.

The amplitude of the wake oscillation is further analyzed introducing the amplitude of the largest Fourier mode along the ζ direction, defined as

$$\Phi^{\clubsuit}_{MAX}=\max_{k}\int_{1}^{L_\zeta} e^{-ik\zeta}\Phi(\rho=0,\zeta)\,d\zeta, \tag{23}$$

where the integration interval is from one electron Debye length downstream of the particle position to the edge of the computational box L_ζ in the downstream direction. This choice of the integration interval is suitable to isolate just the oscillatory region. $\Phi^{\clubsuit}_{MAX}$ measures the amplitude of the wake field as the largest Fourier mode correspond to the oscillation frequency of the wake field. Note that this definition of the wake amplitude is more involved but more rigorous than just simply inspecting the maxima of Figs. 3–4, as the wake field amplitude decreases downstream and the maxima for different Mach numbers and temperature ratios are reached at different positions.

Figure 6 shows the amplitude $\Phi^{\clubsuit}_{MAX}$ for various Mach numbers and temperature ratios. At relatively high ion temperatures (corresponding to a relatively high temperature ratio, $\Theta=10^{-2}$) the amplitude of the wake oscillation decreases monotonically with the Mach number. At lower ion temperatures an increase of the amplitude is present at $M=1$, confirming the results of other linear theories that predict a singularity at $M=1$ for cold ion plasmas [19].

The effect of the collisions between ions and neutral particles is studied in Fig. 7. As the collision rate is increased little effect is observed on the wavelength but the wake field is damped in the downstream direction. For sufficiently high collision frequencies only the first oscillation in the wake remains visible (Fig. 7 for $R=1$).

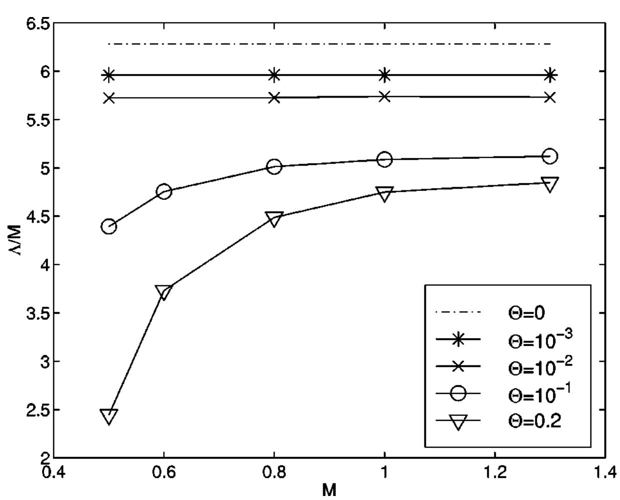

FIG. 5. Wavelength of the wake field as a function of the Mach number for different values of the temperature ratio Θ.

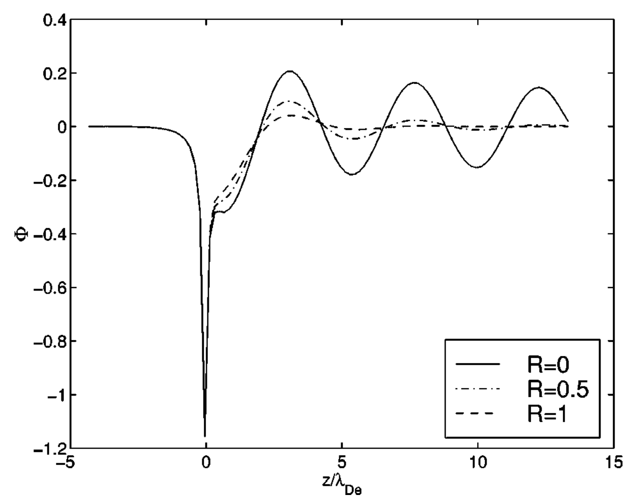

FIG. 7. Perturbation of the normalized potential $\Phi 1$ along the ζ axis for $\Theta=0.01$, $M=0.8$, and for different normalized collision frequencies R.

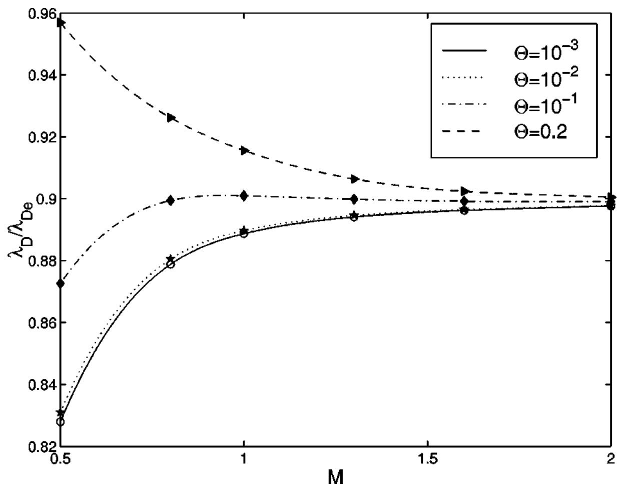

FIG. 8. Radial shielding length λ_D as a function of the Mach number for different values of the temperature ratio Θ.

B. Radial shielding

Simulation works [10] have shown that the screening potential in the radial direction can still be described approximately in terms of the Debye shielding potential

$$\phi_1(r,z=0)\propto\frac{e^{-r/\lambda D}}{r}. \tag{24}$$

The simulations referenced to above have also shown that the correct Debye shielding length λ_D is close to the electron Debye length, and not to the classic linearized Debye length $\lambda_L=(\lambda_{De}^{-2}+\lambda_{Di}^{-2})^{-1/2}$. Recent experiments have proven that indeed the correct shielding length in the radial direction is the electron Debye length [20]. This effect is observed when the ion mean velocity exceeds the ion thermal velocity, as it is in all cases considered here and in most experiments; in such conditions, the ions cannot contribute to the shielding process and the electrons act alone.

Figure 8 confirms this effect. At high Mach numbers the radial shielding length tends to a value close to λ_{De}, common to all temperature ratios. At lower Mach numbers a weak effect of the temperature ratio Θ is observed. Indeed, as the ion speed is reduced, the ion contribution to screening becomes more relevant. Note also that at very low temperature ratios the effect of the ions is to decrease the shielding length, while at higher temperature ratios the effect is opposite. However, at all Mach numbers considered here the effect is rather small.

The main effect is that the shielding length is very close to the electron Debye length and orders of magnitude larger than the linearized Debye length. For example at $\Theta=10^{-3}$, the linearized Debye length is $\lambda_L/\lambda_{De}=0.03$, almost two orders of magnitude smaller than the λ_D observed in experiments and shown in Fig. 8.

V. CONCLUSIONS

A model for the interaction between a plasma and a test particle immersed in it is derived. The model is based on the Boltzmann equation for the plasma species and includes the effect of the collisions with the background neutrals. The model is solved using an expansion in spherical harmonics. To compare with previous theories, the model is reduced to a simplified coupled hyperbolic-elliptic system for the ion density and the electrostatic potential that neglects charge collection by the dust. The final equations of the model are solved numerically.

The model is applied to study the shielding of a test particle in conditions typical of experiments with dusty plasmas. A wake field is observed to propagate downstream of the test particle. A Mach cone with the expected angle is observed. Inside the cone, an oscillatory field is observed with a wavelength predicted by previous theories.

A comparison of the present model with previous theories is successful and suggests that the model can be applied to study the shielding of dust particles, including important effects neglected here: the finite size of the dust particles, the presence of ion and electron currents to the dust, and the existence of an ambient electric field in the sheath region where the dust particles are located.

ACKNOWLEDGMENTS

The author is very grateful to Umberto de Angelis and Vadim Tsytovich for helpful discussions on the topics reported here. Financial support from the Italian Ministry of University and Scientific Research (MURST), under the project MURST 40% Giovani Ricercatori, is gratefully acknowledged.

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